Quasicrystals in Soft Matter

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Overview

Crystals with forbidden symmetry: Quasicrystals are those ordered structures which does not follow the periodic order of atomic arrangement. Until the discovery of quasicrystals in 1980s the crystals were that classification of matter which have 2-,3- or 6- fold rotational symmetries.

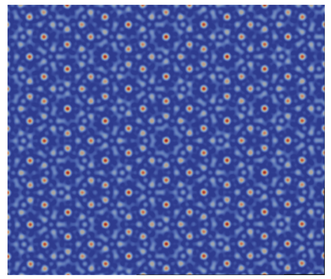

image taken from [3]

Fig. 1: An example of quasicrystal patterns.

But the discovery of quasicrystals expanded the idea of crystals having 5-, 10- and 12- fold symmetries which comes with icosahedral or dodecagonal structure.

Quasicrystals and Soft Matter

The presence of quasicrystals in soft matter has been established during the past decade. Various experiments prove the existance of quasicrystalline structures in soft matter such as ABC star block copolymers and miscelles.

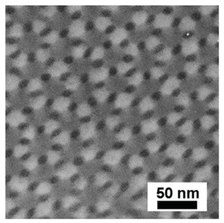

image taken from [1]

Fig. 2: Bright field TEM obtained for ISP terpolymer (polyisoprene (I, black), polystyrene (S, white), and poly(2-vinylpyridine) (P, gray)). The sample used is $I_{1.0}S_{2.7}P_{2.5}$, shows a stable dodecagonal quasicrystal.

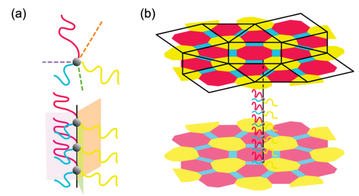

image taken from [1]

Fig. 3: (a) Schematic representation of phase seperation in ABC star terpolymer. (b) When the three components are strongly incompatible the junction tends to align in lines and the tends to form a cylinderical based morphology.

Random Phase Approximation

The Random Phase Approximation is the procedure by which we define the lowest energy state which will give the most stable state for a system. Using this technique we can determine:

- The scattering factor for concentrated polymer solutions like polymer blends and melts which has interaction between the monomers.
- Relation between physical and thermodynamic quantities.

The partition function for a polymer system is given by

$$Z = n_c \int D\rho_q \exp\left(-\frac{1}{2}\left(\sum_q \rho_q \rho_{-q}\left(\frac{V_q\beta}{\Omega} + \frac{1}{\langle\rho_q\rho_{-q}\rangle_0}\right)\right)\right) \quad (1)$$

where $\langle\rho_q\rho_{-q}\rangle_0$ gives $S_0(q)$ which is the density fluctuation for the polymer with out any monomer interaction

2 Component Block Copolymer$(A(BA)_n)$

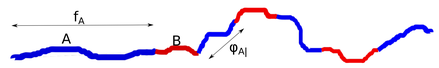

Fig. 4: $A(BA)_n$ block copolymer

Here we consider a block polymer with two different components supposedly A and B. The A monomer has two different fractions in the polymer f_A and ϕ_A and the fraction of monomer B is $\phi_B = 1 - \phi_A$. The structure factor for this polymer is calculated and plotted for a range of wave vectors.

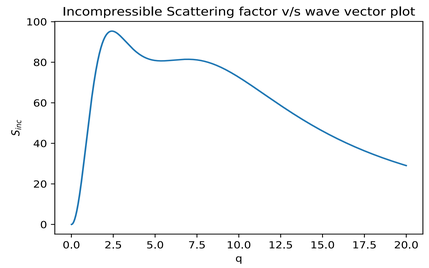

Fig. 5: The structure factor variation for $f_a = 0.3$, $\phi_a = 0.77$ and $n = 8$

The peaks represents the length scales for which it gives a stable structure.
The plot below gives the peak height ratio and wave vector ratio for different fraction of components.

- The stable quasicrystal is expected to be found in the region with wave vector ratio 1.5-2 and height ratio around 1.

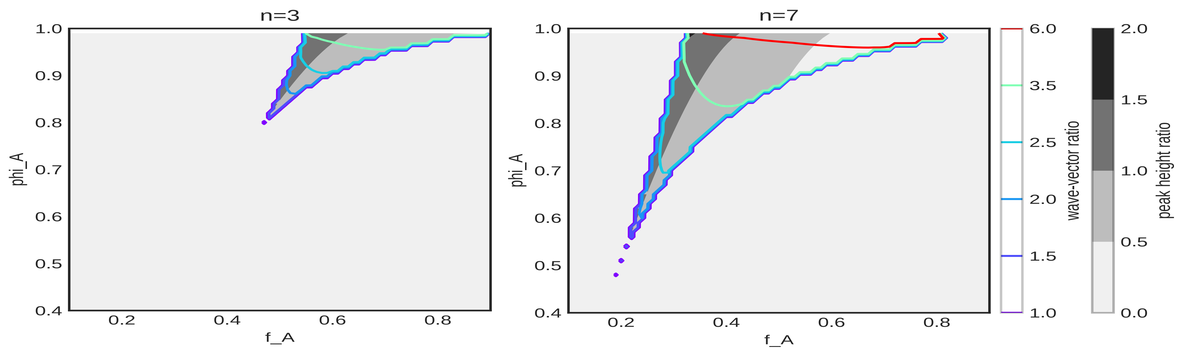

Fig. 6: The cusp region gives region with two peaks in scattering factor when the length fraction of A block is varied

Future work

- To study the stability of quasicrystal in 3-component ABC block copolymer.
- To incoperate polydispersity into the 2 component copolymer.
- To look into the thermodynamics of the block copolymers during phase seperation using Self Consistent Field Theory.

References

[1] Kenichi Hayashida, Tomonari Dotera, Atsushi Takano, and Yushu Matsushita. Polymeric quasicrystal: Mesoscopic quasicrystalline tiling in *abc* star polymers. *Phys. Rev. Lett.*, 98:195502, May 2007.

[2] DJ Read. Calculation of scattering from stretched copolymers using the tube model: a generalisation of the rpa. *The European Physical Journal B-Condensed Matter and Complex Systems*, 12(3):431–449, 1999.

[3] P. Subramanian, A. J. Archer, E. Knobloch, and A. M. Rucklidge. Three-dimensional icosahedral phase field quasicrystal. *Phys. Rev. Lett.*, 117:075501, Aug 2016.