# Friction and adhesion in suspension rheology by constraint counting

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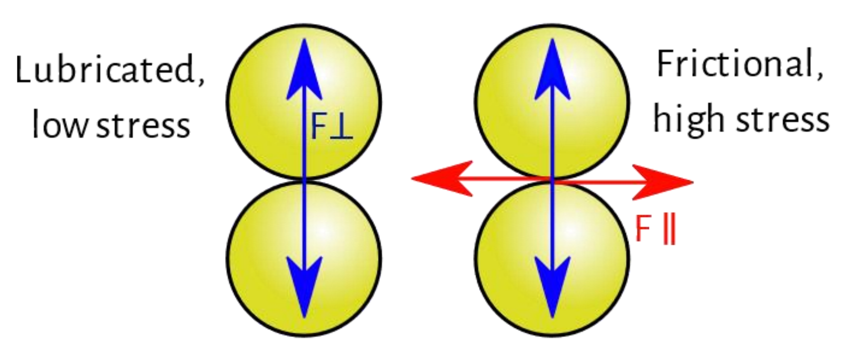
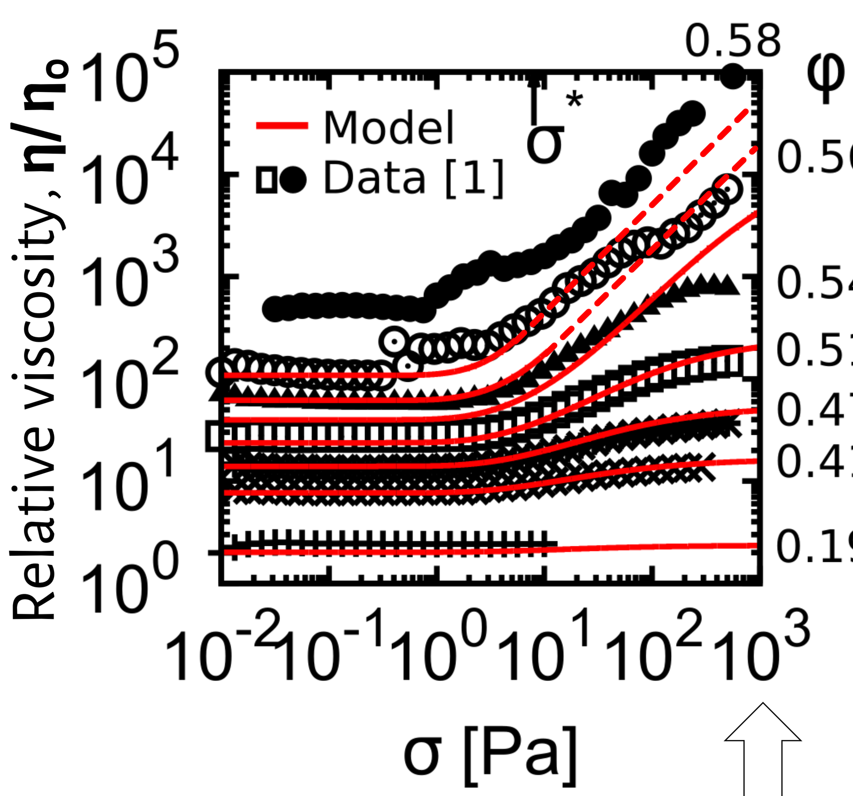
**In theory:** suspensions of hard non-Brownian particles should have a constant viscosity. **In practice:** they are highly non-Newtonian; there must be additional relevant stress scales in these systems.

**Conclusion:** many flow curves are captured by considering constraints (sliding, rolling, ...) at contacts. Rich behaviour arises from the balance of breaking adhesive bonds and making frictional contacts with changing stress.

## Rheology by constraints: revisiting shear thickening and friction

Shear-thickening is driven by the onset of frictional contact at a **critical stress, σ***, when the stabilisation is overcome.



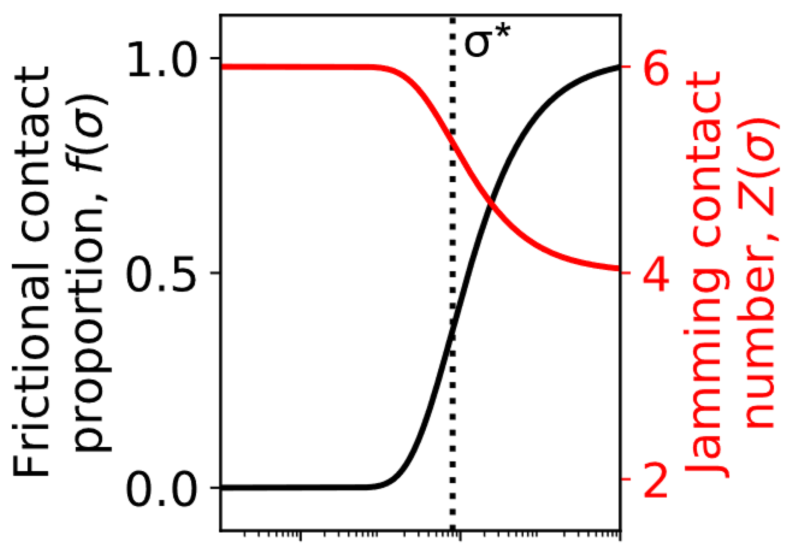




**System**: 4 µm sterically stabilised PMMA in CHB/decalin. C=3.35 & β=0.8

### Recasting Wyart & Cates theory [2]

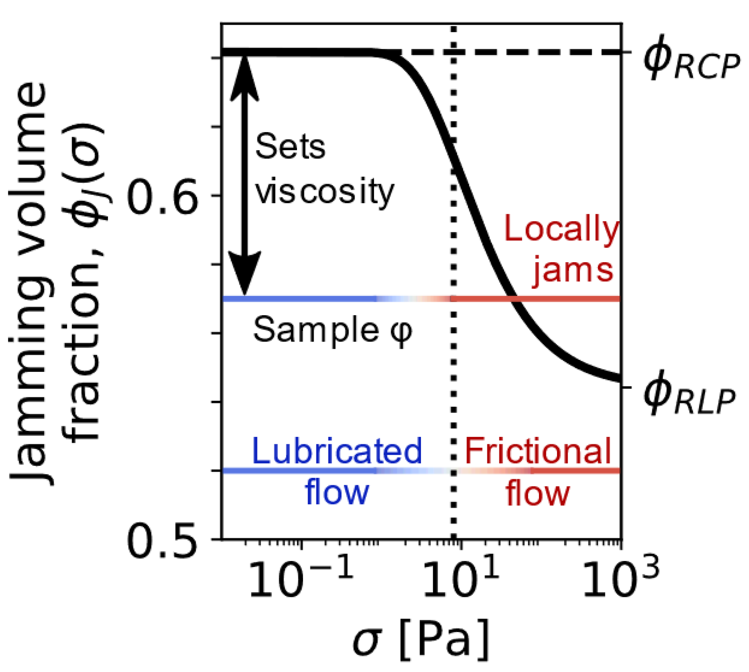
Stress sets the proportion of frictional contacts, ***f*(σ)**:

$$f = \exp\left(-(\sigma/\sigma^\star)^\beta\right)$$




***f*** determines contacts needed to jam, ***Z*(σ)**:

$$Z = 6 - 2f$$




***Z*** controls the jamming volume fraction [3], **$\varphi_J$(σ)**:

$$\phi_J = \frac{Z}{Z + C}$$

Use Kreiger-Dougherty to calculate viscosity, **η(σ,φ)**

$$\eta = \frac{\eta_0}{(1 - \phi/\phi_J)^2}$$

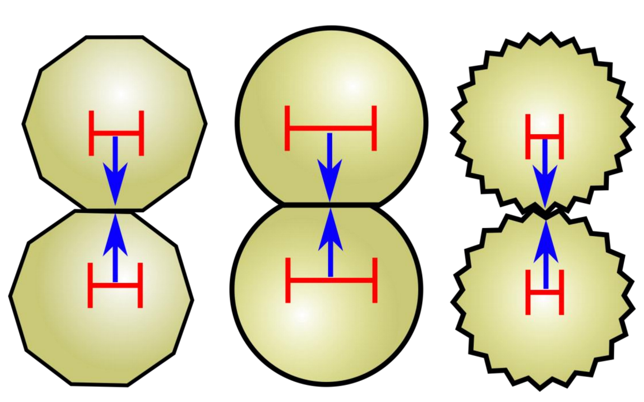
Model flow curves, **η(σ,φ)**

## Introducing another constraint: adhesion

Radial attraction does not add constraints; adhesion, **resisting rolling**, is the relevant effect. The strength will depend on both the interaction and on surface shape.

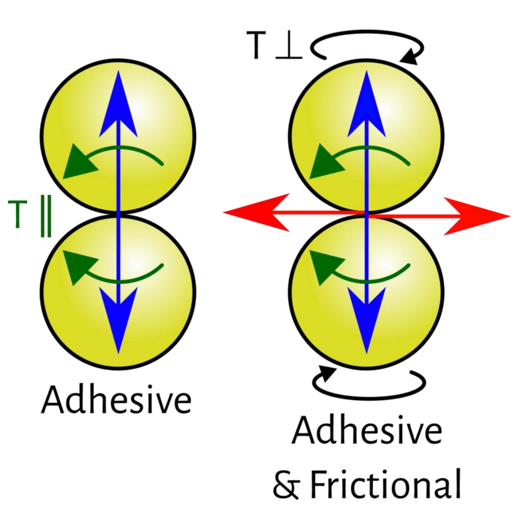
It requires a contact area, which may arise from:

- shape
- deformability, or
- roughness



The proportion of adhesive bonds ***a*** will **decrease** from 1 to 0 with stress above **$\sigma_A$**, the strength of adhesion:

$$a(\sigma) = 1 - \exp(-(\sigma_A/\sigma)^\kappa)$$




Consider complex interactions only in terms of simple constraints, now:

$$Z = 6 - 2a - 2f$$

## References & Funding Acknowledgements

1. Guy, Hermes &Poon, *PRL* (2015). 2. Wyart & Cates, *PRL* (2014). 3. Song, Wang & Makse, *Nature* (2008).

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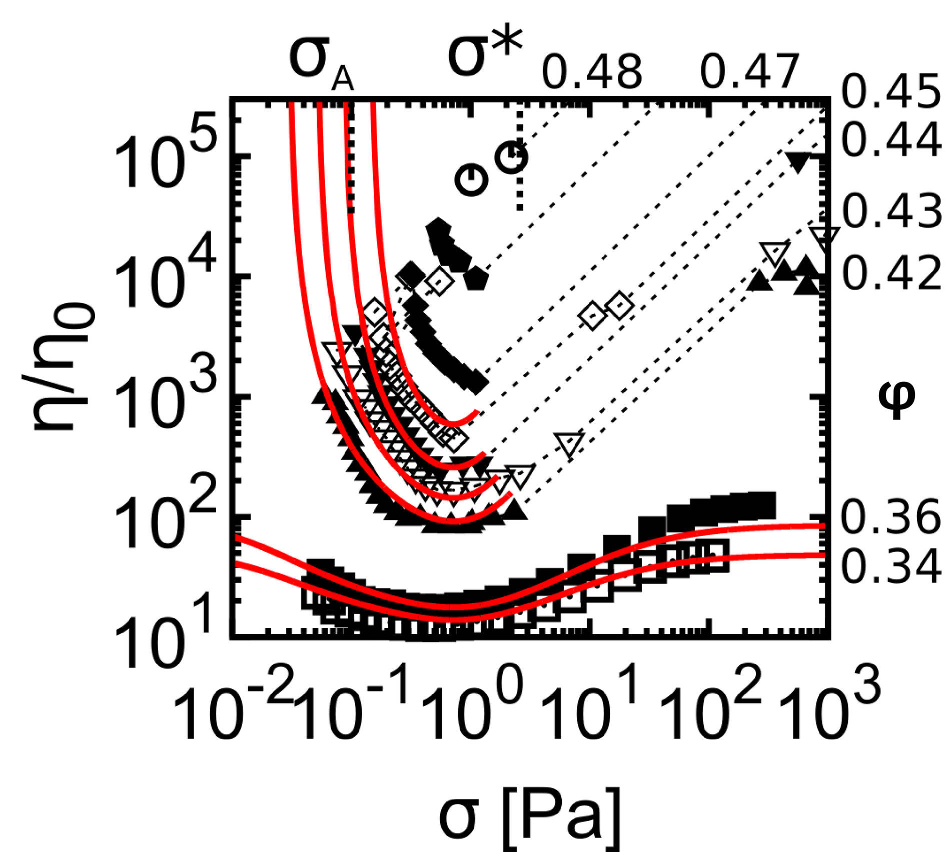
## I. Weak adhesion: $\sigma_A << \sigma^*$

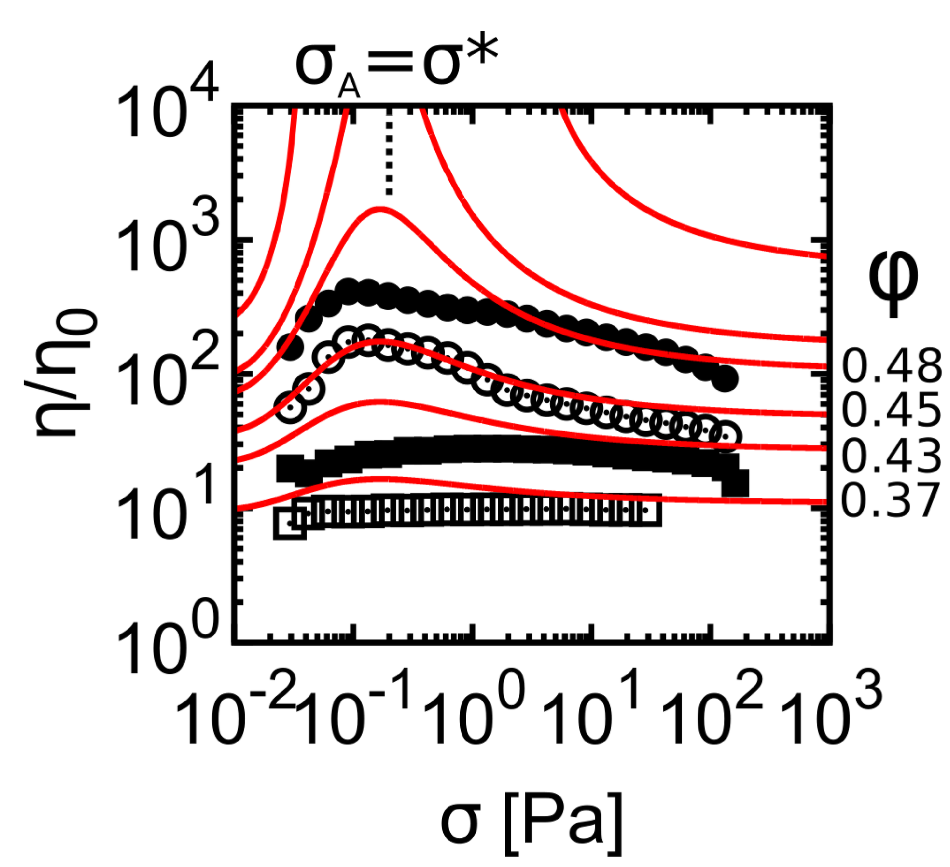



The adhesive bonds are broken before frictional contacts are formed.

Both adhesion and friction introduce two constraints. A yield stress arises at the same φ as Discontinuous Shear Thickening (slope 1 on figure).

**System**: corn-starch in 50wt% water-glycerol. β = 0.8, κ = 0.5 & C = 6.

## II. Intermediate adhesion: $\sigma_A \approx \sigma^*$

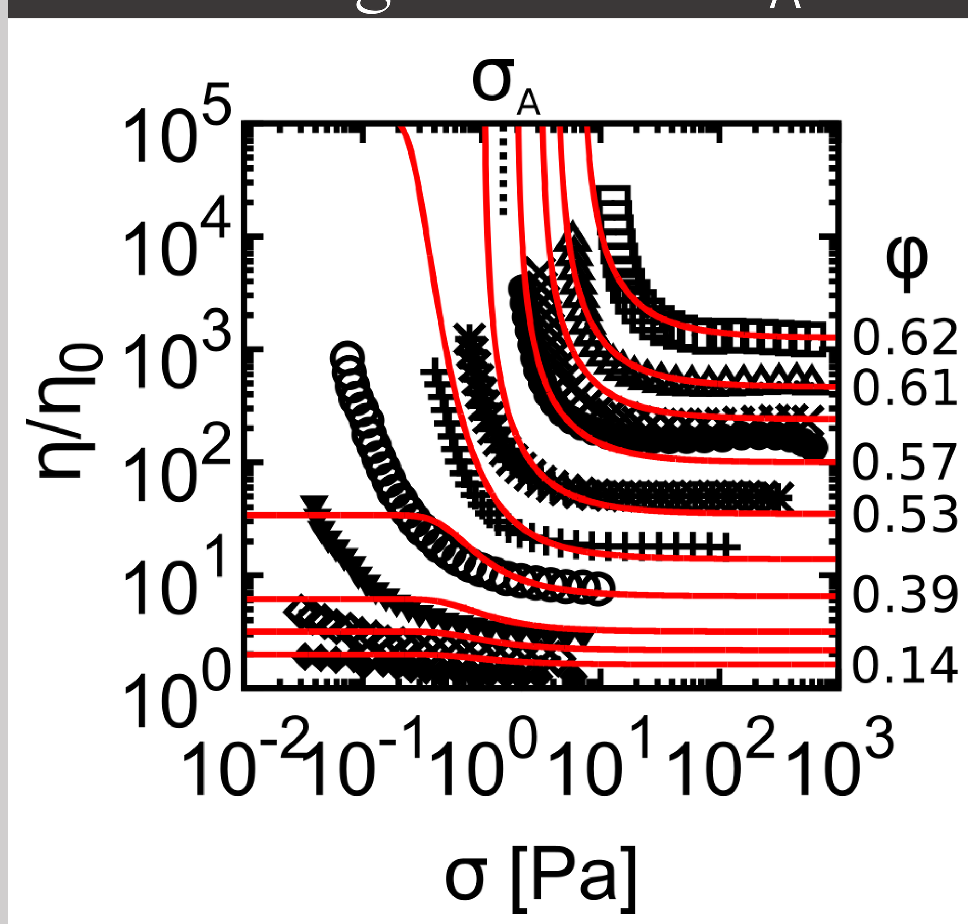



Here the flow depends sensitively on κ/β and the balance of friction and adhesion.

Residual adhesion may explain the prolonged shear thinning seen in granular suspensions and their sensitivity.

**System:** 45 µm sterically stabilised PMMA [1]. β = 1.0, κ = 0.4 & C = 3.6.

## III. Strong adhesion: $\sigma_A >> \sigma^*$




Attraction alone does not explain the yield stress of some granular suspensions.

The yield stress is instead driven by constraints and adhesion, depending strongly on inter-particle friction.

**System**: corn-starch in sunflower oil with lecithin. , κ = 1.1 & C = 2.3.