# Vorticity Banding in Dense Suspensions

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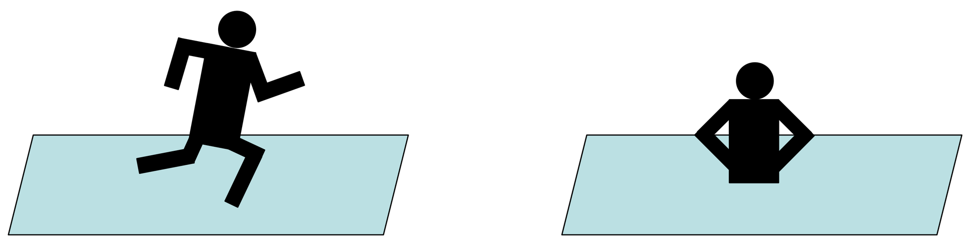
## Introduction

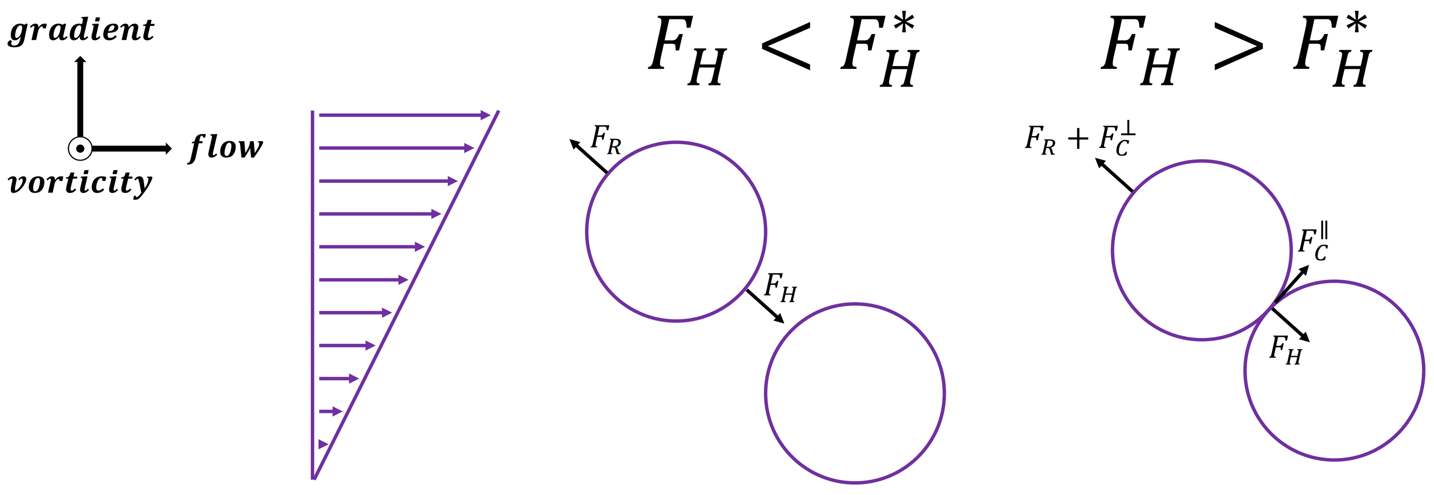
Suspensions, such as corn starch in water, slurries or drilling mud, are systems composed of non-diffusive solid particles suspended in a fluid.



At dense particle concentrations (≳38% by volume for corn starch [1], ≳ 56% for hard spheres [2]), suspensions exhibit rapid, discontinuous increases in stress response to deformations above a critical size, allowing one to run – but not walk – across a bath of concentrated corn starch in water.

## Critical Stress Picture

Recently [2,3], a picture has emerged in which this discontinuous jump in stress response arises due to a transition from a frictionless to a frictional regime in the interactions between particles, when stresses are above a critical stress.




This critical stress is set by stabilising interparticle forces, such as charge or steric stabilisation.

## S-shaped Constitutive Curve

Suspensions of highly frictional particles jam into a solid at lower volume fractions $\phi_J^\mu$ than the jamming fraction $\phi_J^0$ of suspensions of frictionless particles. Wyart and Cates [3] showed that interpolating the jamming fraction $\phi_J$ in the viscosity law

$$\eta = \eta_0(\phi_J - \phi)^{-\nu}$$

between $\phi_J^0$ and $\phi_J^\mu$ as the proportion of interactions involving friction is increased leads to an S-shaped constitutive curve (c.f. fig 3) of steady-state bulk stress versus shear rate for dense enough suspensions, providing one explanation for a discontinuous increase in bulk stress with increasing shear rate.

Stress-controlled systems with S-shaped constitutive curves are typically expected to exhibit flow instabilities leading to heterogeneity along the vorticity direction when $d\sigma/d\dot{\gamma} < 0$ [4], but non-Brownian suspensions are unable to support steady, static bands [5]. Experimental evidence of these instabilities exists [5,6], but until now no particle simulations found banding.

## Vorticity Instability Model

To probe this inconsistency, a constitutive model for the dynamics of suspensions along the vorticity axis was developed:

$$\partial_t \phi + \partial_z(v\,\phi) = 0 \quad \text{mass conservation}$$

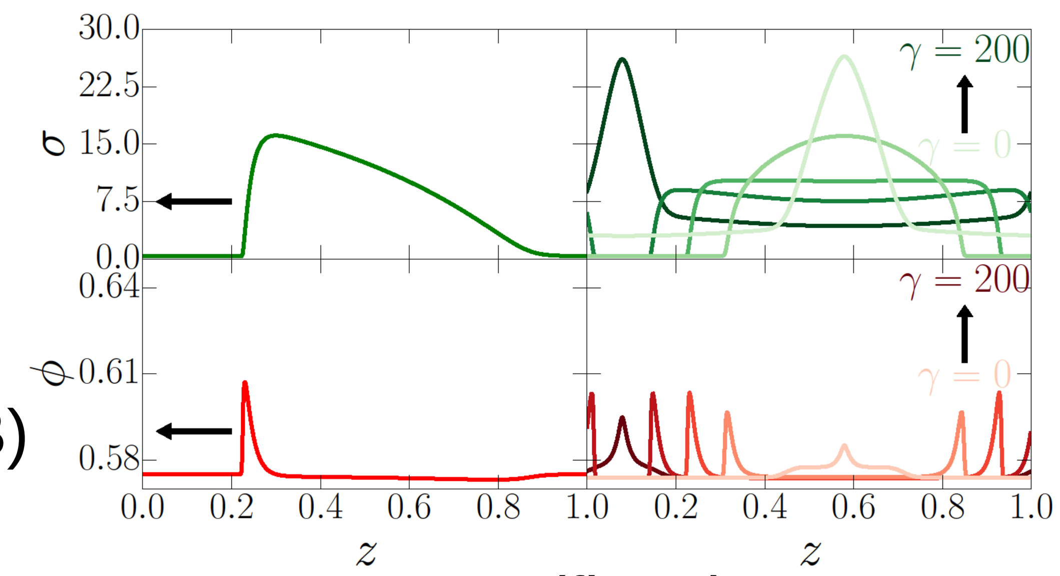
$$\left.\begin{aligned} \partial_t f &= -\frac{\dot{\gamma}}{\gamma_0}[f - f^*(\sigma)] \\ \sigma &= \eta_0(\phi_J(f) - \phi)^{-2}\dot{\gamma} \end{aligned}\right\} \text{Wyart-Cates}$$

$$\partial_z \sigma + \phi\alpha(v - \bar{v}) = 0 \quad \text{force balance}$$

where $f^*(\sigma) = e^{-\sigma_c/\sigma}$ , $\phi_J(f) = f\phi_J^{RLP} + (1-f)\phi_J^{RCP}$ and $\dot{\gamma} = \frac{\bar{\sigma}}{\bar{\eta}}$.

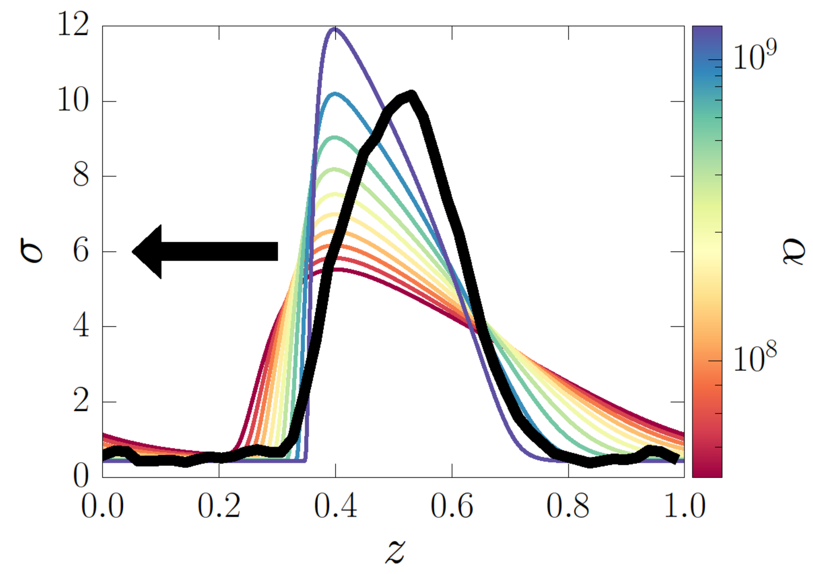
## Results

For sufficiently large systems, our model predicts instabilities leading to two long-time behaviours: travelling bands (TB) (fig 1., left) and standing bands (SB) (fig 1., right).



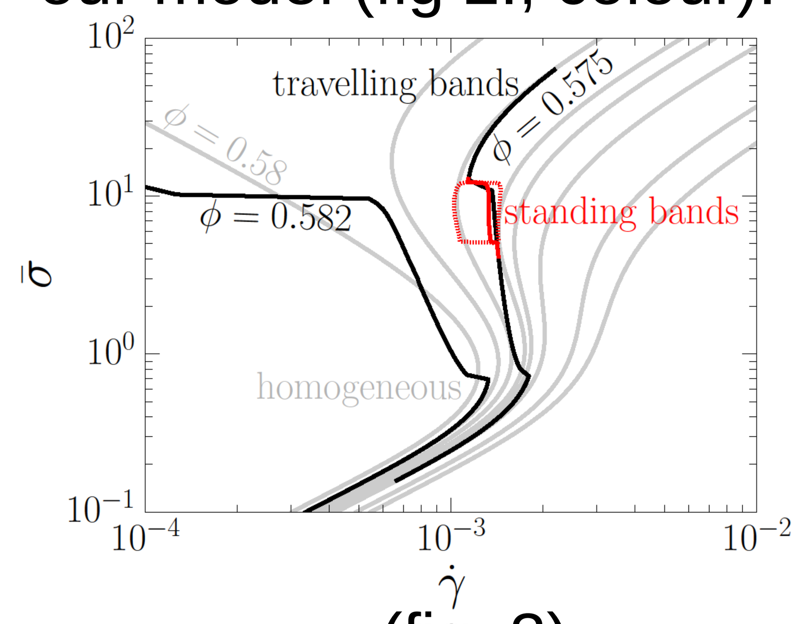

(fig. 1)

We find TB in particle simulations (fig 2., black) with a large vorticity axis, confirming our model's prediction, although the TB profiles differ qualitatively from the TB generated by our model (fig 2., colour).




(fig. 2)

Inhomogeneous flow leads to flow curves that deviate from the homogeneous constitutive curve (fig. 3).




(fig. 3)

## Future Directions

We could try using different boundary conditions to connect with experiments, or modifying the Wyart-Cates terms in search of better fits to particle simulations.

## References

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